PHYSICAL REVIEW E **76**, 055403(R) (2007)

Ferromagnetic behavior in magnetized plasmas

Gert Brodin and Mattias Marklund
Department of Physics, Umeå University, SE-901 87 Umeå, Sweden
(Received 20 September 2007; published 27 November 2007)

We consider a low-temperature plasma within a newly developed magnetohydrodynamic fluid model. In addition to the standard terms, the electron spin, quantum particle dispersion, and degeneracy effects are included. It turns out that the electron spin properties can give rise to ferromagnetic behavior in certain regimes. If additional conditions are satisfied, a homogeneous magnetized plasma can even be unstable. This happens in the low-temperature high-density regime, when the magnetic properties associated with the spin can overcome the stabilizing effects of the thermal and Fermi pressure, to cause a Jeans-like instability.

DOI: 10.1103/PhysRevE.76.055403 PACS number(s): 52.27.Gr, 67.57.Lm

Already in the 1960s, Pines studied the excitation spectrum of quantum plasmas [1], for which we have a high density and a low temperature as compared to normal plasmas. In such systems, the finite width of the electron wave function makes quantum tunneling effects crucial, leading to an altered dispersion relation. Since the pioneering paper by Pines, a number of theoretical studies of quantum statistical properties of plasmas have been done (see, e.g., Ref. [2] and references therein). For example, Bezzerides and DuBois presented a kinetic theory for the quantum electrodynamical properties of nonthermal plasmas [3], while Hakim and Heyvaerts gave a covariant Wigner function approach for relativistic quantum plasmas [4]. Recently, there has been increased interest in the properties of quantum plasmas, e.g., [5–12]. The studies have been motivated by developments in nanostructured materials [13] and quantum wells [14], the discovery of ultracold plasmas [15] (see Ref. [16] for an experimental demonstration of quantum plasma oscillations in Rydberg systems), astrophysical applications [17], or a general theoretical interest. Moreover, it has recently been experimentally shown that quantum dispersive effects are important in inertial confinement plasmas [18]. The list of quantum mechanical effects that can be included in a fluid picture includes the dispersive particle properties accounted for by the Bohm potential [5–9], the zero-temperature Fermi pressure [5–9], spin properties [10–12], and certain quantum electrodynamical effects [19–22]. Within such descriptions [5–11,20–22], quantum and classical collective effects can be described within a unified picture.

In the present paper, we will make use of general equations for spin plasmas that were derived in Ref. [10], and further developed toward the magnetohydrodynamic (MHD) regime in [11]. Exploring the basic set of equations presented in Ref. [11], we demonstrate that the standard plasma behavior can be significantly changed by the electron spin properties, and that the plasma can even show ferromagnetic behavior in the low-temperature, high-density regime. Furthermore, a homogeneous magnetized plasma can actually be unstable, even when the spin degree of freedom is in thermodynamic equilibrium. The instability is due to the magnetic attraction of spins, and the mechanism is conceptually similar to the well-known Jeans instability [23]. Applications of our results to laboratory and astrophysical plasmas are discussed.

Adopting the spin MHD equations put forward in Ref. [11], our plasma is described by the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0, \tag{1}$$

the momentum equation

$$\begin{aligned}\rho\left(\frac{\partial}{\partial t} + \boldsymbol{v}\cdot\nabla\right)\boldsymbol{v} = &-\nabla\left(\frac{B^2}{2\mu_0} - \boldsymbol{M}\cdot\boldsymbol{B}\right) \\ &- \nabla P + \boldsymbol{B}\cdot\nabla\left(\frac{1}{\mu_0}\boldsymbol{B} - \boldsymbol{M}\right) \\ &+ \frac{\hbar^2\rho}{2m_e m_i}\nabla\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right),\end{aligned} \tag{2}$$

and the idealized Ohm's law

$$\frac{\partial \boldsymbol{B}}{\partial t} = \nabla \times (\boldsymbol{v}\times\boldsymbol{B}), \tag{3}$$

where ρ is the plasma density, $\boldsymbol{v}$ the fluid velocity, $\boldsymbol{B}$ the magnetic field, $\boldsymbol{M}$ the magnetization, P the pressure, m_e (m_i) denotes the electron (ion) mass, and $\hbar$ is Planck's constant. In addition to the standard ideal MHD momentum equation, Eq. (2) contains the quantum Bohm potential (which tends to smooth the density profile), as well as magnetization effects due to the electron spin. Equations (1)–(3) should be complemented by an expression for the magnetization, as well as an equation of state for the pressure. In thermodynamic equilibrium, the degree of spin alignment with an external magnetic field is described by the Brillouin functions B_s, where the index s is the spin number. For spin-$\frac{1}{2}$ particles we have $B_{1/2}(\mu_B B/T)=\tanh(\mu_B B/T)$, leading to a corresponding magnetization

$$\boldsymbol{M} = \frac{\mu_B \rho}{m_i}\tanh\left(\frac{\mu_B B}{T}\right)\hat{\boldsymbol{B}}. \tag{4}$$

Here B denotes the magnitude of the magnetic field and $\hat{\boldsymbol{B}}$ is a unit vector in the direction of the magnetic field, $\mu_B = e\hbar/2m_e$ is the Bohr magneton, e is the magnitude of the elementary charge, and T is the temperature given in energy units. In general, the argument of the tanh function can vary if, for example, the magnetic field strength varies. However, when the variations of the magnetic field occur on a time scale shorter than the characteristic spin relaxation time, the degree of alignment can be considered as approximately constant. Since spontaneous spin changes do not occur for single electrons (due to angular momentum conservation), this spin relaxation time is not smaller than the inverse collision frequency, which can be taken as infinite in many applications. This case will be considered for the remainder of this paper, and thus $\tanh(\mu_B B/T) \to \tanh(\mu_B B_0/T)$ in Eq. (4), where the index 0 denotes the unperturbed background value. Furthermore, for low electron temperatures, it is necessary to include the contribution from the zero-temperature Fermi pressure in the equation of state. The equation of state reads

$$\nabla P = c_s^2 \nabla \rho; \tag{5}$$

we note that the ion acoustic velocity c_s includes the contributions from the ion and electron thermal motion, as well as the electron Fermi pressure. Thus we have [22]

$$c_s^2 = v_{ti}^2 + \frac{m_e}{m_i}\left(v_{te}^2 + \frac{3}{5}v_{\mathrm{Fe}}^2\right) \tag{6}$$

where v_{ti} and v_{te} are the (effective) ion and electron thermal velocities [24], whereas v_{Fe} is the electron Fermi velocity [25]. Equations (1)–(6) constitute a closed set that describes the spin-modified quantum MHD equations.

In what follows, we will study the linear modes of this system, with a particular focus on the stability properties. With $\rho=\rho_0+\rho_1$, $\boldsymbol{B}=\boldsymbol{B}_0+\boldsymbol{B}_1$, $\boldsymbol{M}=\boldsymbol{M}_0+\boldsymbol{M}_1$, and $\boldsymbol{v}=\boldsymbol{v}_1$, such that $\rho_1 \ll \rho_0$, $|\boldsymbol{B}_1| \ll |\boldsymbol{B}_0|$, $|\boldsymbol{M}_1| \ll |\boldsymbol{M}_0|$, and $\boldsymbol{B}_0=\boldsymbol{B}_0\hat{\boldsymbol{z}}$, we linearize our equations in the perturbed variables. Assuming that the background quantities are constants, the general dispersion relation can, after a Fourier decomposition, be written

$$(\omega^2 - k_z^2\widetilde{C}_A^2)\{[\omega^2 - k^2\widetilde{C}_A^2 - k_x^2\widetilde{V}_A^2(k)][\omega^2 - k_z^2V_A^2(k)] + k_x^2k_z^2\widetilde{V}_A^4(k)\} = 0, \tag{7}$$

where $\widetilde{C}_A$ is the spin-modified Alfvén velocity given by

$$\widetilde{C}_A = \frac{C_A}{[1 + (\hbar\omega_{pe}^2/2m_ec^2\omega_{ce}^{(0)})\tanh(\mu_B B_0/T)]^{1/2}}, \tag{8}$$

C_A is the standard Alfvén velocity $C_A=(B_0^2/\mu_0\rho_0)^{1/2}$,

$$\widetilde{V}_A^2(k) = V_A^2(k) - \frac{\hbar\omega_{ce}}{m_i}\tanh\left(\frac{\mu_B B_0}{T}\right), \tag{9}$$

and

$$V_A^2(k) = c_s^2 + \frac{\hbar^2k^2}{4m_im_e}. \tag{10}$$

Here $\omega_{pe}=(\rho_0e^2/\varepsilon_0m_em_i)^{1/2}$ is the plasma frequency, and $\omega_{ce}^{(0)}$ is the electron cyclotron frequency associated with the external magnetic field (i.e., with the contribution to B_0 from the spin sources excluded). The relation between the full electron cyclotron frequency $\omega_{ce}=eB_0/m_e$ and $\omega_{ce}^{(0)}$ is given by $\omega_{ce}=\omega_{ce}^{(0)}+(\hbar\omega_{pe}^2/m_ec^2)\tanh(\mu_B B_0/T)$. We stress that $\widetilde{V}_A$, which to some extent can be considered as an effective acoustic velocity, may be imaginary for a strongly magnetized plasma due to the spin contribution, a fact that will be explored in some detail below.

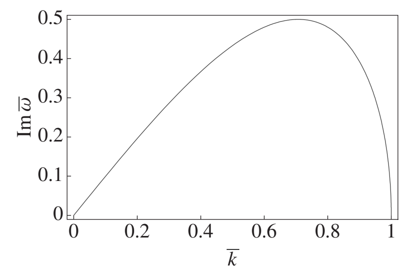

FIG. 1. Growth rate Im $\bar{\omega}$ as a function of $\bar{k}$ obtained from the dispersion relation (11). The relation (11) is of the form $\omega = k(b^2k^2-a^2)^{1/2}$, and we use the normalization $\bar{\omega}=|b|\omega/|a|^2$ and $\bar{k}=|b|k/|a|$. We note that we have a maximal growth rate for the wave number $k_{\max}=|a|/(\sqrt{2}|b|)=[2m_em_i(|P_{\mathrm{sp}}|-P_m-P)/\rho_0\hbar^2]^{1/2}$, corresponding to a maximal growth rate $\gamma_{\max}=(m_em_i)^{1/2}(|P_{\mathrm{sp}}|-P_m-P)/\rho_0\hbar$. Furthermore, although (11) is a linear dispersion relation, the present instability shows similarities to the modulational instability.

In deducing Eq. (7) we have assumed that the spin orientation has reached the thermodynamic equilibrium state in response to the external magnetic field. This ensures that there is no free energy stored in the spin degree of freedom, and as a consequence it turns out that the shear Alfvén mode described by the first factor of (7) is always stable, since clearly $\widetilde{C}_A$ is always real. This is related to the fact that this particular mode has no density perturbations. By contrast, the second factor, describing the fast and slow magnetosonic modes, does not necessarily predict stability. The reason is that the electrons carry spin, and thus to some extent they behave as single magnets. Just like magnets or gravitating matter, the electrons may thus attract each other, leading to an exponentially growing density, similar to the gravitational Jeans instability. Naturally, electrostatic repulsion among the electrons could in principle act as a strong counteracting force to this scenario. However, within the low-frequency MHD limit, ions and electrons move together, and thus the Coulomb force does not provide a stabilizing mechanism. To shed some further light on the stability properties, we consider propagation perpendicular to the external magnetic field, which is the geometry that leads to instability most easily. For the case $\boldsymbol{k}=k\hat{\boldsymbol{x}}$, Eq. (7) reduces to

$$\omega = k\left[\frac{C_A^2}{1 + (\hbar\omega_{pe}^2/2m_ec^2\omega_{ce}^{(0)})\tanh(\mu_B B_0/T)} + c_s^2 + \frac{\hbar^2k^2}{4m_im_e} - \frac{\hbar\omega_{ce}}{m_i}\tanh\left(\frac{\mu_B B_0}{T}\right)\right]^{1/2}. \tag{11}$$

The condition for instability is thus that the last negative term of (11) dominates over all the others. Under this assumption, we have depicted the growth rate as a function of k in Fig. 1.

The necessary and sufficient instability condition can thus be written as

$$P_{\rm sp} + P_{\rm m} + P + \frac{\rho_0 \hbar^2 k^2}{4m_e m_i} < 0, \tag{12}$$

where the total pressure $P_{\rm tot} = P_{\rm sp} + P_m + P$ consists of the effective spin pressure $P_{\rm sp} = -(\rho_0 \hbar \omega_{ce}/m_i)\tanh(\mu_B B_0/T)$, which is the only negative pressure term and therefore the source of the instability, the magnetic pressure P_m, and the particle pressure $P = n_0 m_i c_s^2$, containing both the thermal and Fermi pressure parts. Furthermore, the magnetic pressure P_m is given by

$$P_m = \frac{\rho_0 C_A^2}{1 + (\hbar \omega_{pe}^2 / 2m_e c^2 \omega_{ce}^{(0)})\tanh(\mu_B B_0/T)}. \tag{13}$$

Thus a necessary (although not sufficient) condition for instability is

$$\lambda > \lambda_c \equiv \pi \left(\frac{\hbar}{eB_0 \tanh(\mu_B B_0/T)} \right)^{1/2}, \tag{14}$$

which means that the instability is stabilized for short wavelengths $\lambda = 2\pi/k$, similar to the Jeans instability. The stabilizing influence for short wavelengths stems from the Bohm potential. Furthermore, the partial instability condition

$$|P_{\rm sp}| > P \tag{15}$$

means that a finite pressure also may lead to stabilization. We note from Eq. (6) that a low temperature is not necessary to satisfy this condition, since the zero-temperature Fermi velocity contributes to c_s^2 and thereby to P. However, for a sufficiently strong magnetic field, clearly (15) can be satisfied. Finally, the last part of the instability condition reads

$$|P_{\rm sp}| > P_m, \tag{16}$$

which means that the magnetic pressure also acts as a stabilizer. For a given magnetic field, this condition may be satisfied for a sufficiently high density. However, increasing the density means that the Fermi velocity is increased, which may lead to a violation of (15). To simultaneously satisfy (15) and (16), and thereby to satisfy (12), it is required that the second term in the denominator of the right side of (13) be larger than unity, i.e.,

$$\omega_{ce}^{(0)} < \frac{\hbar \omega_{pe}^2}{2m_e c^2} \tanh\left(\frac{\mu_B B_0}{T} \right). \tag{17}$$

Since the spin cannot contribute much to the unperturbed field unless the temperature is small enough to allow a significant alignment, this condition in turn requires

$$1 \lesssim \frac{\hbar^2 \omega_{pe}^2}{2m_e c^2 T}. \tag{18}$$

For temperatures small enough to satisfy (18), the spin contribution to the unperturbed field dominates over the external field, and the plasma thus shows ferromagnetic behavior. In contrast to a normal ferromagnet, however, the density variations are not restricted, which render possible the instability discussed above. However, plasmas with the required background parameters are not easy to produce, as we can see from the following examples. First, if we choose a high-density plasma as in inertial fusion experiments, $\rho \sim 10^6\ \mathrm{kg/m^3}$, ferromagnetic behavior occurs for temperatures $T \lesssim 10^4 - 10^5$ K, as described by the inequality (18). In this regime the Alfvén velocity can differ much from the standard Alfvén velocity, as given by (8). If, in addition, we want the Jeans-like instability to occur, the most severe condition to satisfy is (15), which requires temperatures $T \lesssim 20$ K for standard laboratory field strengths. Until a few years ago, the only known plasmas where such low temperatures could be found were solid state plasmas, which do not fit into the MHD-like model used here. However, recently gaseous plasmas with ultralow temperatures $T \lesssim 10^{-3}$ K have been constructed with the aid of Rydberg atoms [15,16]. Unfortunately, the combined requirement of a reasonably high density, Eq. (18), rules out the spin instability described above in such a laboratory setting.

In addition to laboratory applications, the theories described above could be adopted for astrophysical purposes [17]. In magnetar atmospheres, the strong magnetic field makes it possible to satisfy the conditions (15) even for a relativistic temperature. In that case, we should adopt the theory to a pair plasma [12] rather than an ion-electron plasma. Furthermore, for white dwarf stars, the high density causes the condition (18) to be satisfied.

[1] D. Pines, J. Nucl. Energy, Part C **2**, 5 (1961).
[2] D. Kremp, M. Schlanges, and W.-D. Kraeft, *Quantum Statistics of Nonideal Plasmas* (Springer, Berlin, 2005).
[3] B. Bezzerides and D. F. DuBois, Ann. Phys. (N.Y.) **70**, 10 (1972).
[4] R. Hakim and J. Heyvaerts, Phys. Rev. A **18**, 1250 (1978).
[5] G. Manfredi, Fields Inst. Commun. **46**, 263 (2005).
[6] F. Haas, G. Manfredi, and M. R. Feix, Phys. Rev. E **62**, 2763 (2000).
[7] F. Haas, Phys. Plasmas **12**, 062117 (2005).
[8] L. G. Garcia, F. Haas, L. P. L. de Oliveira, and J. Goedert, Phys. Plasmas **12**, 012302 (2005).
[9] P. K. Shukla, Phys. Lett. A **352**, 242 (2006).
[10] M. Marklund and G. Brodin, Phys. Rev. Lett. **98**, 025001 (2007).
[11] G. Brodin and M. Marklund, New J. Phys. **9**, 277 (2007).
[12] G. Brodin and M. Marklund, e-print arXiv:0708.0020, Phys. Plasmas (to be published).
[13] H. G. Craighead, Science **290**, 1532 (2000).
[14] G. Manfredi and P.-A. Hervieux, Appl. Phys. Lett. **91**, 061108 (2007).
[15] W. Li, P. J. Tanner, and T. F. Gallagher, Phys. Rev. Lett. **94**, 173001 (2005).
[16] R. S. Fletcher, X. L. Zhang, and S. L. Rolston, Phys. Rev. Lett. **96**, 105003 (2006).
[17] A. K. Harding and D. Lai, Rep. Prog. Phys. **69**, 2631 (2006).

[18] S. H. Glenzer *et al.*, Phys. Rev. Lett. **98**, 065002 (2007).
[19] M. Marklund and P. K. Shukla, Rev. Mod. Phys. **78**, 591 (2006).
[20] E. Lundström *et al.*, Phys. Rev. Lett. **96**, 083602 (2006).
[21] G. Brodin, M. Marklund, B. Eliasson, and P. K. Shukla, Phys. Rev. Lett. **98**, 125001 (2007).
[22] J. Lundin, J. Zamanian, M. Marklund, and G. Brodin, Phys. Plasmas **14**, 062112 (2007).
[23] L. Herrera and N. O. Santos, Phys. Rep. **286**, 53 (1997).
[24] For temperatures where the thermal velocity is much lower than the Fermi velocity, the square of the effective thermal velocity for species j $(j=i,e)$ is given by $v_{tj}^2=C(T/\hbar n^{1/3})$, where C is a dimensionless constant of order unity. For temperatures much higher than the Fermi velocity we may use $v_{tj}^2=T/m_j$.
[25] The electron Fermi velocity is given by $v_F=\hbar(3\pi^2 n)^{1/3}/m_e$.